Time Domain Green's Functions for Lossy and Dispersive Multilayered Media

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***Abstract*—We have introduced a fast method of calculating the time domain Green's functions for multilayered media. In this paper, we demonstrate the use of this method to compute the scalar potential Green's function for a multilayer lossy dispersive medium on a PEC ground. The strength of the method lies in obtaining the Green's function for many source-to-field distances ρ and time instances t simultaneously. It only takes 6 min 28 s to compute $100 \times 336 = 33\ 600$ space time Green's function points in Matlab on a Pentium III 867 MHz processor with 1 GB of RAM for a multilayered lossy dispersive medium.**

***Index Terms*—Absorbing media, dispersive media, Green function, nonhomogeneous media, time domain analysis.**

I. Introduction

THERE has been a lot of interest in the computation of the Green's functions for planarly layered media. This is because the Green's functions describe the mathematical relationship between the source and the field such that integral equations can be used to solve for the currents in the layered media problem. Important applications of such integral equation solutions are signal integrity of high speed digital interconnects and analysis of microstrip circuits.

Work by Michalski and Mosig focused on obtaining the frequency domain dyadic Green's functions for planarly-stratified, multilayered media [1]. The objective was to use the mixed potential integral equation (MPIE) to analyze 3-D objects embedded in layered media.

Recent work by Okhmatovski and Cangellaris improves on the efficiency of evaluating the frequency domain Green's function for large electrical distances between the source and the field [2]. The work in [2] also avoids the extraction of surface wave poles, which would be difficult for multilayered media.

Due to more recent applications that require analysis of wideband phenomena, like digital signals and short pulses, fast EM analysis in the time domain is becoming increasingly important. The common fast method of calculating the time-domain Green's function for layered media has been based on full wave complex images and FFTs [3]. The method of complex images [3] loses accuracy for a certain distance range [2]. It is also less useful for multilayered media.

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We have introduced an efficient method of calculating the Green's functions for the time-domain electric field integral equation (TDEFIE) for multilayered media [4]. The method is outlined as follows.

1) Complex frequency is used so that the surface wave poles are lifted off the real k_ρ axis [5]. The integration in k_ρ can be carried out on the real k_ρ axis. This is an advantage for multilayered media as pole extraction would be difficult.
2) Half-space extraction is used so that the integrand of the nonhalf-space portion decays exponentially on the real k_ρ axis, even for the worst case where both the source and the field points are on the surface [6]. This speeds up the evaluation of the Green's function even for large source-to-field distances. The nonhalf-space portion is evaluated using the fast Hankel transform (FHT). The FHT yields results for many source-to-field points simultaneously.
3) Branch cut integration is used to integrate the half-space portion. The integrand of the branch cut integral decays exponentially so convergence of the integral is quite fast.
4) FFT is used in the complex frequency plane to compute the time-domain Green's functions from the frequency domain. Methods employing the free-space time-domain Green's function would have difficulty evaluating dispersive media such as FR-4.

In the previous paper [4], we demonstrated the use of our method in solving for the Green's functions for lossless, nondispersive multilayered media. In this paper, we extend and demonstrate the use of our method in calculating the Green's function for lossy, dispersive, multilayered media. This is important because most signal integrity problems occur on printed circuit boards such as FR-4, which is lossy and dispersive. Throughout this paper, only the time domain Green's function for the scalar electric potential Φ will be discussed. The time domain Green's function for the vector magnetic potential $\bar{A}$ can be evaluated in a similar manner.

II. Mathematical Formulation

For a multilayered medium with both source and field points on the top layer, the frequency domain Green's function G_v is given by

$$G_v(\rho,\omega) = -\frac{j}{4\pi\varepsilon_0}\int_0^\infty dk_\rho \frac{1}{k_z}\left[k_\rho + \frac{k_0^2 R^{TE} + k_z^2 R^{TM}}{k_\rho}\right] \times J_0(k_\rho \rho) \quad (1)$$

where $k_z = \sqrt{k_0^2 - k_\rho^2}$. R^{TE} and R^{TM} are the TE and TM reflection coefficients respectively due to a plane wave incident on the multilayered medium. R^{TE} and R^{TM} can be calculated from equations found in [7] and [8]. The source is taken to be at the origin. The ω in (1) is complex so as to avoid singularities on or near the real k_ρ axis.

The time-domain convolution of the Green's function and the time-domain source function can be obtained from the frequency domain Green's function $G_v(\rho,\omega)$. Let $\omega = \omega' + j\omega''$ for complex frequency and $X(\omega)$ be a frequency-domain source function. Noting that the time-domain convolution must be real

$$\int_{-\infty}^{\infty} G_v(\rho, t-t')\,\rho_s(0,t')\,dt' = \frac{e^{-\omega'' t}}{\pi}\,\mathrm{Re}\left[\int_0^{\infty} d\omega' e^{j\omega' t} X(\omega' + j\omega'')\,G_{A,v}(\rho, \omega' + j\omega'')\right].$$

$G_v(\rho,\omega)$ can be separated into the half-space and nonhalf-space portion

$$G_v(\rho,\omega) = G_v^{(H)}(\rho,\omega) + G_v^{(N)}(\rho,\omega)$$

where

$$G_v^{(H)}(\rho,\omega) = -\frac{j}{4\pi\varepsilon_0}\int_0^{\infty} dk_\rho \frac{1}{k_z}\left[k_\rho + \frac{k_0^2 R_{01}^{TE} + k_z^2 R_{01}^{TM}}{k_\rho}\right] \times J_0(k_\rho\rho) \quad (2)$$

$$G_v^{(N)}(\rho,\omega) = -\frac{j}{4\pi\varepsilon_0}\int_0^{\infty} dk_\rho \frac{1}{k_z} J_0(k_\rho\rho) \times \left[\frac{k_0^2\left(R^{TE} - R_{01}^{TE}\right) + k_z^2\left(R^{TM} - R_{01}^{TM}\right)}{k_\rho}\right]. \quad (3)$$

R_{01}^{TE} and R_{01}^{TM} are the reflection coefficients for TE and TM polarized waves respectively incident on the boundary between layers 0 and 1 where layer 1 is infinitely thick. Equation (2) is evaluated using branch cut integration. This is to speed up the evaluation of the integrals as the integrands decay exponentially along the branch cuts. Equation (3) is evaluated using the fast Hankel transform (FHT). The FHT transforms the Sommerfeld integral into a convolution integral such the FFT can be used in its evaluation. The FHT yields results for many source-to-field points simultaneously. The use of complex frequency shifts the poles off the real k_ρ axis allowing the integration in (3) to be evaluated without pole extraction.

III. Modeling of Lossy Dispersive Media

The lossy dispersive media has to have relative permittivity characteristics that obey the laws of causality. Fig. 1 shows a two-layer lossy dispersive media on a PEC ground, which is analyzed in this paper. The bottom material is chosen as FR4. The top material is less lossy than FR4, and is henceforth referred to as material A.

The FR4 material is modeled according to (3) found in [9]. The less lossy material A was modeled using [9, Eq. (5)]

$$\varepsilon_r(f) = \varepsilon'_\infty + \frac{\Delta\varepsilon'}{m_2 - m_1}\log_{10}\left(\frac{f_2 + jf}{f_1 + jf}\right)$$

where $\varepsilon'_\infty = 2.179$, $\Delta\varepsilon' = 4.015 \times 10^{-2} + j1.681 \times 10^{-3}$, $f_1 = 1$ Hz, $f_2 = 10$ GHz, $m_1 = 0$ and $m_2 = 10$.

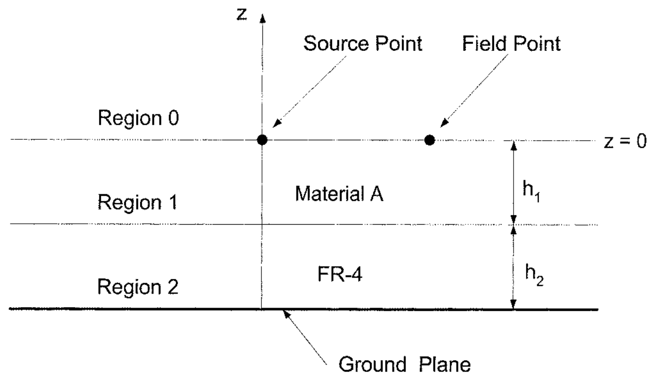

Fig. 1. Two-layer lossy dispersive media.

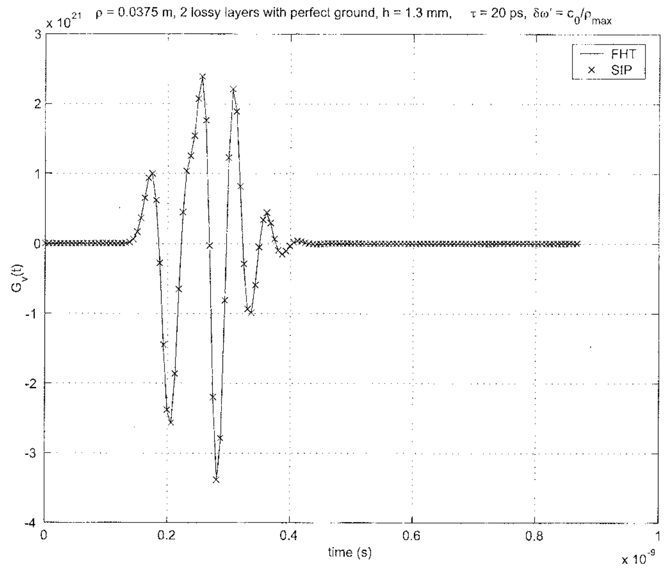

Fig. 2. Validation of the accuracy of the method described in the paper.

IV. Numerical Results

Fig. 2 shows the comparison of the accuracy between $\int_{-\infty}^{\infty} G_v(\rho, t-t')\,\rho_s(0,t')\,dt'$ for the two-layer lossy dispersive media on the PEC ground plane generated with complex frequency and real frequency. The comparison function was computed using real frequency with the nonhalf-space portion numerically integrated along the Sommerfeld integral path (SIP). The layered media is as shown in Fig. 1. The source-to-field distance $\rho = 3.75$ cm. The height of each layer $h = 1.3$ mm. The source function was a Gaussian pulse with a pulse width $\tau = 20$ ps. The agreement between the two solutions is very good. This validates the method we described in Section II for the analysis of lossy dispersive multilayered media.

Fig. 3 shows the comparison of $\int_{-\infty}^{\infty} G_v(\rho, t-t')\,\rho_s(0,t')\,dt'$ for the same lossy dispersive media shown in Fig. 1 with that of a lossless, nondispersive media. The nondispersive and lossless media has first layer as $\varepsilon_r = 2.2$ and second layer as $\varepsilon_r = 4.3$. ρ, h and τ are the same as for Fig. 2. The lossy dispersive media "dampens" the Green's function as time progresses. Fig. 3 shows that errors result in the late time if lossy dispersive media is assumed to be lossless and nondispersive.

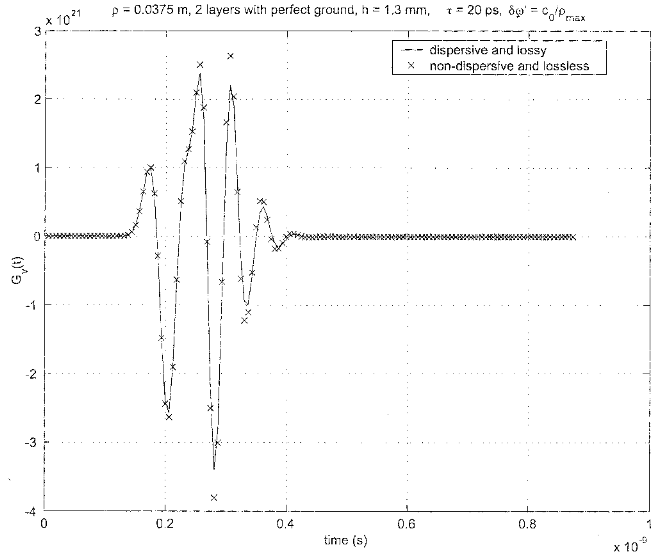

Fig. 3. Showing the difference between lossy dispersive case and lossless nondispersive case.

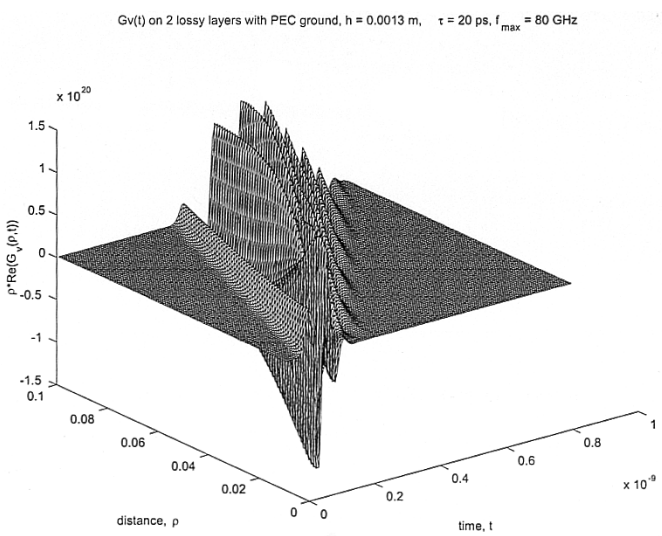

Fig. 4. Three-dimensional mesh of 100 points of ρ and 336 points of t.

The strength of the method described in Section II lies in obtaining the Green's function for many source-to-field distances ρ and time instants t simultaneously through the FHT and the FFT respectively. Fig. 4 shows the 3-D mesh of $\rho \int_{-\infty}^{\infty} G_v(\rho, t-t') \rho_s(0,t')\, dt'$ as a function of ρ and t. The geometry of the structure is as shown in Fig. 1. Computation of the Green's function for 100 points in ρ and 336 points in t in Matlab took 6 min 28 s on a Pentium III 867 MHz processor with 1 GB of RAM. The computation time would be further reduced if the program is written in C or fortran. In contrast, if the same number of points were calculated on the same machine using real frequency and without any fast algorithms, the calculation would take over 10 h in Matlab.

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